RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FIRST SEMESTER EXAMINATION, DECEMBER 2012

FIRST YEAR

Date : 19/12/2012 **Mathematics (General)**

Time : 11.00 am – 2.00 pm Paper : I Full Marks : 75

Use separate answer-book for each group

Group – A

(Answer **any five** from Q. no. 1 to Q. no. 8)

1. (i) If $\sin\alpha+\sin\beta+\sin\gamma=\cos\alpha+\cos\beta+\cos\gamma=0$ then prove that

   $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\sin^2\alpha+\sin^2\beta+\sin^2\gamma=\frac{3}{2}$. 3

   (ii) Express $(a+ib)^{p+iq}$ in the form $A+iB$, where *a, b, p, q* are real numbers. 2

2. State the fundamental theorem of Classical Algebra.

   Show that the roots of the equation $\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}=\frac{1}{x}$ (where $a > b > c > 0$) are real. 1+4

3. (i) The sum of the two roots of the equation $x^3+a_1x^2+a_2x+a_3=0$ is zero.

   Show that $a_1a_2-a_3=0$. 3

   (ii) If ∝, β, γ be the roots of the equation $x^3-px^2+qx-r=0$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$. 2

4. Solve the following equation using Cardan's method: 5

   $x^3+6x^2-12x+32=0$

5. a) Show that a skew-symmetric determinant of order 3 vanishes. 2

   b) Show that

   $$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$$ 3

6. Find the inverse of the matrix

   $$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}$$

   and use it to solve the system of equations $x-y+z=1,\ x+y+2z=0,\ 2x-y+3z=2$ 3+2

7. a) Find the rank of the matrix

   $$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -4 & 7 \\ -1 & -2 & -1 & -2 \end{pmatrix}$$ 3

b) If $A=\begin{bmatrix} 1 & -2 & 3 \\ 4 & 0 & -5 \\ -3 & 2 & 4 \end{bmatrix}$ find B such that $2A^T+3B=4I_3$. 2

8. a) Solve the following system of equations using Cramer's rule: 3

$$3y-x+z=4$$
$$2z-y=0$$
$$x-y+z=2$$

b) If a and b are real numbers, prove that $\cos\left(i\log\frac{a-ib}{a+ib}\right)=\frac{a^2-b^2}{a^2+b^2}$. 2

Group – B

(Answer **any five** questions taking at least **two** from each unit)

Unit-I

9. a) Let A,B,C be three subsets of an universal set U. If $A\cap C=B\cap C$ and $A\cap C'=B\cap C'$, then prove that $A = B$. (Here P' means complement of P). 3

b) Given $A = \{1, 2\}$, $B = \{3, 4\}$. Prove that $A\times B\neq B\times A$. 2

10. a) Show that the mapping $f:\mathbb{Q}\to\mathbb{Q}$ defined by $f(x)=2x+5$ is one-one and onto, where $\mathbb{Q}$ is the set of all rational numbers. Hence find f^{-1} . 3

b) If $f:X\to Y$ and $g:Y\to Z$ be two injective mappings then prove that the composite $g\circ f:X\to Z$ is injective. 2

11. a) Prove that the set of even integers forms an additive group. 3

b) Let $(G,\bullet)$ be a group. If $a^2 = e$ for all $a\in G$, prove that $(G,\bullet)$ is a commutative group. 2

12. Prove that the Ring of Matrices $M=\left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}: a,b\in\mathbb{Q}\right\}$ is a field with respect to usual Matrix addition and multiplication. [$\mathbb{Q}$ = set of Rational Numbers] 5

Unit-II

13. Find the nature of the real quadratic form $x^2+y^2-z^2+2xy-2yz+2zx$. 5

14. i) If α, β, γ be linearly independent vectors of a vector space V over the field of real numbers then show that $\alpha+\beta$, $\alpha-\beta$, $\alpha-2\beta+\gamma$ are also linearly independents vector in V. 3

ii) Find a basis of the real vector space R^3 containing the vectors (1, 1, 2) and (3, 5, 2). 2

15. State Cayley Hamilton's theorem and use it to find A^{-1} where $A=\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$. 1+4

16. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$. 5

Group – C

(Answer **any five** from Q. no. 17 to Q. no. 24)

17. A function $f: R \to R$ is defined in the following way:

$$f(x) = -x, \text{ when } x \le 0$$
$$= x, \text{ when } 0 < x < 1$$
$$= 2 - x, \text{ when } x \ge 1$$

Show that it is continuous at $x = 0$ and $x = 1$. Check the derivability of f at $x = 0$ and $x = 1$. 5

18. Show that $\sqrt{2}$ is not a rational number. Also find $f'(1)$ if $f(x) = 2|x| + |x-2| \; \forall x \in R$. 2+3

19. If $f(x, y) = xy\left(\dfrac{x^2 - y^2}{x^2 + y^2}\right)$, when $x^2 + y^2 \neq 0$

$= 0$, when $x^2 + y^2 = 0$

show that $f_{xy}(0,0) \neq f_{yx}(0,0)$. 5

20. i) If $u = \log r$ and $r^2 = x^2 + y^2 + z^2$, prove that $r^2\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right) = 1$. 3

ii) State Euler's theorem on homogeneous functions of three variables. 2

21. i) Prove that all points of the curve $y^2 = 4a\left\{x + a\sin\left(\dfrac{x}{a}\right)\right\}$ at which the tangent is parallel to the x-axis lie on a parabola. 3

ii) Find the condition that the conics $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ shall cut orthogonally. 2

22. If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$ then show that $\rho^{-2/3} + \rho^{-2/3} = (2a)^{-2/3}$. 5

23. i) Find the pedal equation of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. 4

ii) State Leibnitz's theorem on the derivative of the product of two functions of x. 1

24. i) Find the nth derivative of $e^{ax}\sin bx$. 2

ii) If $y = \sin(m\sin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$ where $y_n = \dfrac{d^n y}{dx^n}$. 3